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Perron-Frobenius Theory – Some Extensions and Applications

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Abstract

The Perron-Frobenius theory on nonnegative matrices was introduced by Perron and Frobenius in the beginning of the 20th century. Since its construction the Perron-Frobenius theory has been developed and constituted a basic Linear Algebra tool to study and solve problems arising from applications in discretization of Differential and Integral Equations, Markov chains, Economics, Biosciences, etc. The class of M-matrices, which was introduced and studied in the meantime, appears in many of the aforementioned applications. Also, some classes of splittings (regular, weak regular, nonnegative, etc.) were proposed for the solution of large linear algebraic systems of equations by iterative methods.

The Perron-Frobenius theory of nonnegative matrices was extended to matrices which have some negative entries, by D. Noutsos [*Linear Algebra Appl., 412 (2006), no 2–3, 132–153*]. Some properties which give information, when a matrix possesses a Perron-Frobenius eigenpair, were presented and proved. This class of matrices is associated to eventually nonnegative matrices, namely matrices whose powers become and remain nonnegative. The class of Perron-Frobenius splitting was proposed and studied for the solution of linear systems by classical iterative methods. Some properties of the type of Stein-Rosenberg theorem were extended to the class of Perron-Frobenius splittings by D. Noutsos [*Linear Algebra Appl., 429 (2008), 1983–1996*].

Linear differential systems $\dot{x}(t) = Ax(t)$, $A \in \mathbf{R}^{n,n}$, whose solutions become and remain nonnegative, were studied by D. Noutsos and M. J. Tsatsomeros [*SIAM J. Matrix Anal. Appl., 30 (2008), no 2, 700–712*]. Initial conditions that result to nonnegative states are shown to form a convex cone that is related to the matrix exponential e^{tA} and its eventual nonnegativity.

Further extension of the Perron-Frobenius theory of nonnegative matrices to certain complex matrices was proposed and proved by D. Noutsos and R. S. Varga [*Linear Algebra Appl., 437 (2012), 1071–1088*].

Recently, B. Iannazzo and D. Noutsos, in a forthcoming paper, have considered an extension of the Perron-Frobenius theory to matrices obtained by a suitable scaling (with positive and negative entries) applied to an M-matrix. This problem appears, for instance, in the study of Algebraic Riccati Equations arising in fluid queues, where one is interested in the invariant subspaces of the matrix

$$H = \begin{bmatrix} A & -B \\ -C & -D \end{bmatrix},$$

obtained by changing the sign of the last m rows of an M-matrix.

Key words: Nonnegative Matrices, Perron-Frobenius Theory, Eventually Nonnegative Matrices, M-Matrices, Riccati Equation.