

## Sensitivity of the Domain Decomposition Method to Perturbation of the Transmission Conditions

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### Abstract

Nowadays Domain Decomposition (DD) method is one of the common tools to construct preconditioner to solve 3D Helmholtz equation, especially in geophysical applications. There are numerous papers devoted to construction of optimal transmission conditions to improve convergence of the DD. However, these researches are focused on the differential statements and no perturbation is typically assumed. Whereas solution of the 3D Helmholtz equation requires the use of the numerical methods such as finite differences of finite elements, thus a numerical error is introduced in the transmission conditions as a result of numerical approximation. Moreover, in some cases it is worth using different numerical methods in adjoint subdomains, which makes the considered perturbations nonsymmetric.

In this paper, a simplest 1D Helmholtz equation was considered and the perturbation of the Dirichlet-to-Neumann map based transmission conditions are considered. It was proved, that if the perturbation is symmetric (the same numerical methods and discretizations are used in the adjoint subdomains) the numerical solution converges to the true solution for almost all practically meaningful cases. However, if the perturbation is nonsymmetric, i.e. different numerical methods are used, different discretizations are applied, different approximations of the boundary operators are utilized, the numerical solution converges, but not to the solution of the original problem. In this case, an irreducible error presents, which linearly depends on the perturbation of the transmission conditions.

The research was done under financial support of the Russian Foundation for Basic Research grants no. 13-05-00076, 13-05-12051, 14-05-00049, 14-05-93090, 14-01-31340, fellowship SP-150.2012.5 of the President of the Russian Federation, and integration projects of SB RAS 127 and 130.

*Key words:* Domain decomposition, Helmholtz equation.