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Generalizations and Modifications of Iterative Methods for Solving Large Sparse Indefinite Linear Systems

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Abstract

An overview of generalizations and modifications of iterative methods for solving large sparse indefinite linear systems with both symmetric and nonsymmetric coefficients matrices.

Keywords: iterative methods, large sparse indefinite linear systems, generalizations and modifications of iterative methods, Arnoldi process, GMRES, SYMMLQ/SYMMQR, Lanczos SYMMLQ/SYMMQR.

Frequently, when computing numerical solutions of partial differential equations, we need to solve systems of very large sparse linear algebraic equations of the form

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

where $\mathbf{A}$ is a given $n \times n$ matrix, $\mathbf{b}$ the given righthand side vector, and we seek a numerical solution vector $\mathbf{x}$ or a good approximation of it. Particularly for large linear systems arising from partial differential equations in three dimension, well-known direct methods based on Gaussian elimination may become prohibitively expensive in terms of both computer memory and computer time. On the other hand, iterative methods may avoid these difficulties.

While the conjugate gradient (CG) method (and variations of it) may work well, for linear systems with a symmetric positive definite (SPD) coefficient matrices $\mathbf{A}$, the choice of a suitable iterative method is *not* at all clear, when the linear system has a symmetric indefinite coefficient matrix.

We discuss a variety of iterative methods that are based on the Arnoldi Process for solving large sparse symmetric indefinite linear systems. We describe the SYMMLQ and SYMMQR methods, as well as, generalizations and modifications of them. Then, we cover the Lanczos/MSYMMLQ and Lanczos/MSYMMQR methods, which arise from a double linear system. We present some pseudocodes for these algorithms.

Finally, we mention some additional generalizations and modifications of iterative methods for solving large sparse symmetric and nonsymmetric indefinite systems of linear equations such as GMRES, MGMRES, MINRES, LQ-MINRES, QR-MINRES, MMINRES, and others.