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A Meshfree Method with Fundamental Solutions for Inhomogeneous Elastic Wave Problems

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Abstract

We consider the numerical solution of the inhomogeneous Cauchy-Navier equations of elastodynamics, assuming time-harmonic variation for the displacement field $\mathbf{U}(x,t) = \mathbf{u}(x)e^{-i\omega t}$ of an isotropic material with Lamé constants λ and μ and density ρ. The resulting elliptic PDE, posed in a bounded simply connected domain Ω is coupled with Dirichlet boundary conditions and solved trough a meshfree method, based on the Method of Fundamental Solutions (MFS).

$$\begin{cases} \mu\Delta\mathbf{u} + (\lambda+\mu)\nabla(\nabla\cdot\mathbf{u}) + \rho\omega^2\mathbf{u} = \mathbf{f} & \text{in } \Omega \\ \mathbf{u} = \mathbf{g} & \text{on } \Gamma \end{cases}$$

In particular, an extension, from the scalar [1, 2] to the vector case, of the MFS is applied and the displacement field $\mathbf{u}$ is approximated in terms of a linear combination of fundamental solutions (Kupradze tensors) of the corresponding homogeneous PDE with different source points and test frequencies. The applicability of the numerical method is justified in terms of density results [3]. The high accuracy and the convergence of the proposed method will be illustrated through 2D numerical simulations. Convex and non-convex domains and different sets of boundary data and body forces will be considered. Interior elastic wave scattering problems will also be addressed.

Key words: Method of Fundamental Solutions, Inhomogeneous BVP, Elastic Wave Propagation.

References

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