Conference in Numerical Analysis 2014 (NumAn 2014)
September 2-5, 2014
Chania,Greece

On the numerical modelling and solution of multi-asset Black-Scholes equation based on Generic Approximate Sparse Inverse Preconditioning

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Abstract

One of the most important topics in the area of financial mathematics is the study of the multi-asset Black-Scholes equation for the pricing of options. While there is a closed-form solution in one dimension for pricing European vanilla options, in higher dimensions the finite difference method allows the consideration of a wider range of parameters (coefficients of the partial differential equation, initial and boundary conditions). Hence, research efforts have been directed towards finding accurate prices for options with two or more underlying assets. In this paper, we present a fourth order accurate discretization scheme for the numerical solution of Black-Scholes equation in two space variables.

The purpose of this work is to derive fourth order accurate option pricing methods while maintaining low computational complexity. For the space discretization we use a fourth order finite difference scheme combined with Richardson's extrapolation method while for the time integration high order Backward Differences along with fourth order Gauss-Legendre Runge-Kutta scheme was used. The resulting sparse linear system of algebraic equations is solved by preconditioned iterative techniques based on generic approximate sparse inverses. Herewith, the Preconditioned Induced Dimension Reduction (PIDR(s)) method in conjunction with Generic Approximate SParse Inverse (GenAspI) is used for the efficient solution of the sparse linear systems. The GenAspI is computed through an incomplete factorization of the coefficient matrix to a predefined sparsity pattern acquired from Powers of Sparsified Matrices (PSM's), thus handling any sparsity pattern.

Numerical results are presented along with discussions for the proposed schemes in order to highlight the applicability and efficiency for solving the Black-Scholes equation in two space variables. The implementation issues of the proposed method are also discussed.

Key words: Multi-Asset Black-Scholes equation, high order finite difference schemes, sparse linear systems, generic approximate sparse inverses, preconditioned induced dimension reduction method.