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Fokas method and Kelvin transformation applied to potential problems in non convex unbounded domains.

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Abstract

In this presentation Fokas integral method is combined with Kelvin transformation to develop a new method for solving Dirichlet or Neumann problems in non-convex unbounded domains. A key aspect in Fokas method is the coupling of all boundary values in one equation, which has been termed global relation. Through this, any missing data on a boundary value problem can be derived, as Dassios and Fokas have shown. On the other hand, Kelvin transformation preserves harmonicity, and thus, by applying it to an exterior potential problem, the solution of the equivalent interior problem can be established, in the domain which is the Kelvin image of the original exterior one. In the present work, these two methods have been employed in order to derive integral representations for the Dirichlet and the Neumann problem in a non-convex domain which is the Kelvin image of an equilateral triangle. The proposed methodology for the case of a Neumann exterior problem is given below. Physically, this could be explained as the construction of a potential for a vector field of which the effect of its normal derivative is known along its boundary that is assumed to be the image of an equilateral triangle under the Kelvin transformation.
First, we apply the Kelvin inversion and thus the corresponding Neumann data on the boundary of the equilateral triangle are obtained. By then employing the Neumann to Dirichlet map, the Dirichlet data on the perimeter of the triangle are extracted. Subsequently, an integral representation of the solution of the Neumann problem in the interior of the triangle is accomplished. Applying again the Kelvin transformation to the attained Dirichlet data we derive the corresponding Dirichlet data on the initial boundary. By employing Kelvins 2-D theorem, we eventually obtain an integral representation of the solution of the Neumann problem in the given exterior non convex domain. Furthermore, we derive the Neumann to Dirichlet map for every Fourier component of some arbitrary data. This way, we provide a basis for representing the Dirichlet data on the boundary and thus we can obtain an integral representation of the solution of a large class of potential problems regarding non convex domains, encounted in many fields of science and engineering, that they would not be possible otherwise.
Alternatively, in the case of a Dirichlet problem, we pursue the proposed methodology modified appropriately to obtain analogous results.

Key words: Fokas method, Kelvin inversion integral representation , potential problems