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Solving the Fredholm integral equation of the second kind by global spline quasi-interpolation of the kernel

P. Sablonnière[a] and D. Barrera[b]

[a] INSA & IRMAR, Rennes, France

[b]Department of Applied Mathematics, University of Granada, Granada, Spain

Paul.Sablonniere@insa-rennes.fr,dbarrera@ugr.es

Abstract

For solving the linear Fredholm integral equation of the second kind

$$u(x) = f(x) + \int_a^b k(x,t)u(t)dt,$$

we propose to approximate the kernel by *tensor products* or *blending sums* of univariate spline quasi-interpolants (abbr. QI). These QIs have the general form $Qf := \sum_{j \in J} c_j(f)B_j$, where the B_j are B-splines defined on some partition of $[a,b]$ and the coefficients $c_j(f)$ are linear functionals based on values of the function f on some finite subset $\mathbf{S} := \{s_j, j \in J\}$ of the interval $I := [a,b]$. Thus, the kernel will be approximated

1. either by the *tensor product* of two univariate quasi-interpolants Q_1 and Q_2 in the variables x and t:

$$k(x,t) \approx (Q_1 \otimes Q_2)k(x,t) = \sum_{i,j} K_{i,j} B_i(x) B_j(t),$$

where the coefficients $K_{i,j}$ are linear combinations of values $k(s_i, s_j)$, for $(i,j) \in J \times J$,

2. or by the *continuous blending sum* of the two univariate quasi-interpolants Q_1 and Q_2:

$$\begin{aligned} k(x,t) &\approx (Q_1 \oplus Q_2)k(x,t) := (Q_1 \otimes Id + Id \otimes Q_2 - Q_1 \otimes Q_2)k(x,t) \\ &= \sum_i \widetilde{k}_i(t) B_i(x) + \sum_j \overline{k}_j(x) B_j(t) - \sum_{i,j} K_{i,j} B_i(x) B_j(t), \end{aligned}$$

where the functions $\widetilde{k}_i(t) = c_i(k(.,t))$ (resp. $\overline{k}_j(x) = c_j(k(x,\cdot))$) are linear combinations of left sections $k(s_k, t)$ (resp. right sections $k(x, s_\ell)$) of the kernel.

When substituting these approximate (degenerate) kernels in the Fredhom equation, we get two types of approximate solutions:

- $u(x) = f(x) + \sum X_i B_i(x)$ in the tensor-product case,
- $u(x) = f(x) + \sum X_i B_i(x) + \sum Y_j \overline{k}_j(x)$ in the continuous blending case,

The vectors of variables X_i and Y_j are then solutions of systems of linear equations. The two methods can be used with any type of spline QIs, although only C^1 quadratic and C^2 cubic splines will be considered.

Key words: Fredholm equation, quasi-interpolation, tensor product, boolean sum.