Conference in Numerical Analysis 2014 (NumAn 2014)
September 2-5, 2014
Chania,Greece

Error Estimates for the Standard Galerkin-Finite Element Method for the Shallow Water Equations

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Abstract

We consider a simple initial-boundary-value problem for the shallow water equations on a finite interval, and also the analogous problem for a symmetric variant of the system that we justify for small-amplitude solutions. Assuming smoothness of solutions we discretize these problems in space using the standard Galerkin-finite element method and prove L^2-error estimates for the semidiscrete problem for quasiuniform and uniform meshes. In particular we show that the semidiscretization with piecewise linear, continuous functions on a uniform mesh posseses optimal-order $\mathcal{O}(h^2)$ L^2-error estimates. We also examine time-stepping of the semidiscrete problems with three explicit Runge-Kutta methods (the Euler, improved Euler, and the Shu-Osher scheme), and prove L^2-error estimates for the resulting full discretizations that are of optimal order in the temporal variable. We also discuss the cases of periodic and absorbing boundary conditions.

Key words: Shallow water equations, fully discrete Galerkin methods, error estimates.