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The WR-HSS Methods for Non-Self-Adjoint Positive Definite Linear Differential Equations and Applications to the Unsteady Discrete Elliptic Problem

Xi Yang[a],

[a]Dept. Math., Nanjing University of Aeronautics and Astronautics,
Nanjing 210016, Jiangsu, P.R. China
yangxi@lsec.cc.ac.cn

Abstract

We consider the numerical methods for non-self-adjoint positive definite linear differential equations,

$$\mathcal{L}(x) = B\,\dot{x} + A\,x = q, \quad x(0) = x_0, \tag{1}$$

with B being Hermitian and A being non-Hermitian positive definite, and their corresponding applications to the unsteady discrete elliptic problem, which is derived from spatial discretization of the unsteady elliptic problem with Dirichlet boundary condition, i.e.,

$$\begin{cases} \frac{\partial u}{\partial t} - \nabla \cdot [a(x)\nabla u(x)] + \sum_{j=1}^{d} \frac{\partial}{\partial x_j}(p(x)u(x)) = f(x),\ u(x,0) = u_0(x),\ x \in \Omega \\ \text{Dirichlet Boundary Condition.} \end{cases} \tag{2}$$

Taking into account the idea of Hermitian/skew-Hermitian splitting (HSS) in [1], we establish a class of waveform relaxation iteration methods based on the HSS splitting of the non-self-adjoint positive definite linear operator $\mathcal{L}$, i.e., WR-HSS methods. We analyze these WR-HSS methods with the help of Fourier Transform. Similarly to the HSS methods for solving linear algebraic equations, we find that the WR-HSS methods are unconditionally convergent to the solution of (1). In addition, we derive the upper bound of the contraction factor of the WR-HSS methods which is only dependent on the Hermitian part of $\mathcal{L}$. Finally, the applications of these WR-HSS methods to the unsteady discrete elliptic problem demonstrate their effectiveness and the corresponding theoretical results.

Key words: elliptic problem, Hermitian/skew-Hermitian splitting, waveform relaxation.

References

[1] Z.-Z. Bai, G.H. Golub and M.K. Ng, Hermitian and skew-Hermitian splitting methods for non-Hermitian positive definite linear systems, *SIAM J. Matrix Anal. Appl.*, 24(2003), 603-626.