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Structured Strong Linearizations obtained from Fiedler Pencils with Repetition

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Abstract

Let $P(\lambda)$ be a matrix polynomial of degree $k \geq 2$ whose coefficients are $n \times n$ matrices with entries in a field $\mathbb{F}$. A matrix pencil $L(\lambda) = \lambda L_1 - L_0$, with $L_1, L_0 \in M_{kn}(\mathbb{F})$, is a *linearization* of $P(\lambda)$ if there exist two unimodular matrix polynomials (i.e. matrix polynomials with constant nonzero determinant), $U(\lambda)$ and $V(\lambda)$, such that

$$U(\lambda)L(\lambda)V(\lambda) = \begin{bmatrix} I_{(k-1)n} & 0 \\ 0 & P(\lambda) \end{bmatrix}.$$

Beside other applications, linearizations of matrix polynomials are used in the study of the polynomial eigenvalue problem. For each matrix polynomial $P(\lambda)$, many different linearizations can be constructed but, in practice, those sharing the structure of $P(\lambda)$ are the most convenient from the theoretical and computational point of view, since the structure of $P(\lambda)$ often implies some symmetries in its spectrum.

In this talk we present $nk \times nk$ matrix pencils obtained from the family of Fiedler pencils with repetition, introduced by S. Vologiannidis and E. N. Antoniou (2011), preserving the structure of $P(\lambda)$, when $P(\lambda)$ is symmetric, skew-symmetric or T-alternating. Under certain conditions, these pencils are strong linearizations of $P(\lambda)$. These linearizations are companion forms in the sense that, if their coefficients are viewed as k-by-k block matrices, each $n \times n$ block is either 0_n, $\pm I_n$, or $\pm A_i$, where A_i, $i = 0, \ldots, k$, are the coefficients of $P(\lambda)$.

Key words: Structured linearization, Fiedler pencils with repetition, matrix polynomial, companion form, polynomial eigenvalue problem.