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Numerical Solution Of Optimization Problems for Semilinear Elliptic Equations with Discontinuous Coefficients and Solutions

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Abstract

Mathematical models of optimization for systems with distributed parameters (described by equations of mathematical physics) are the most difficult class of problems in optimization, especially for nonlinear optimal control problems. By ”non-linear optimization problems” for equations of mathematical physics we understand those in which the mapping $g \to u(g)$ from the set of admissible controls U to the space of states W is a nonlinear. Particular formulations of optimization problems for distributed parameter systems depend substantially on whether the controls enter into the free terms of the equations of state or in the equation coefficients and on whether linear or nonlinear PDEs describe the states of the control systems. Linear control systems with sufficiently smooth input data and control state functions have been most thoroughly studied and nonlinear optimization problem have been least studied to this day. Especially interesting from theoretical and practical points of view are optimal control problems in which the states are described by nonlinear PDEs with discontinuous coefficients and the solutions can be discontinuous due to the character of the physics process under study.

Before solving optimal control problems numerically, they have to be approximated by problems of a simpler nature, specifically, by ”finite-dimensional problems”. One of the most convenient and universal techniques for finite-dimensional approximation as applied to optimization problems is the grid method. Also relevant is the question of the development of efficient numerical methods for solving constructed finite-dimensional grid optimal control problems, which requires effective procedures for calculating the gradient of the minimized functional.

In this work we consider optimization problems for processes described by semilinear partial differential equations of elliptic type with discontinuous coefficients and solutions (with imperfect contact matching conditions), with controls involved in the coefficients. Finite difference approximations of optimization problems are constructed. For the numerical implementation of finite optimization problems differentiability and Lipshitz-continuity of the grid functional of the approximating grid problems are proved. Effective procedures for calculating gradients of minimized functionals using the solutions of direct problems for the state and adjoint problems are obtained.

Key words: semilinear elliptic equations, optimization problem, numerical method.

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