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Splitting methods based on Approximate Matrix Factorization and Radau-IIA formulas for the time integration of advection diffusion reaction PDEs.

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Abstract

A family of methods for the time integration of evolutionary Advection Diffusion Reaction Partial Differential Equations (PDEs) semi-discretized in space is introduced. The methods are obtained by combining a splitting $J_h = \sum_{j=1}^{d} J_{h,d}$ of the Jacobian matrix J of the resulting ODE -where h is a small positive parameter related to the spatial resolution, such as the mesh-width- and a number of inexact Newton Iterations applied to the two-stage Radau IIA method. The overall process reduces the storage and the algebraic cost involved in the numerical solution of the multidimensional linear systems to the level of one-dimensional linear systems with small bandwidths.

The local error of AMF-Radau methods when applied to semi-linear equations is described. From here, since the order in time is at most three, some specific methods considering up to three inexact Newton Iterations are selected. Furthermore, linear stability properties for the selected methods are established, in such a way that the wedges of stability depend on the number of terms d considered in the splitting J_h. In particular, $A(\alpha_d)$-stability is shown for $1 \leq d \leq 4$, where $\alpha_d := \min\{\frac{\pi}{2}, \frac{\pi}{2(d-1)}\}$, and $A(0)$-stability for any $d \geq 1$.

Numerical experiments on 2D and 3D problems are presented, and they show that the methods compare well with standard classical methods in parabolic problems and can also be successfully used for advection dominated problems whenever some diffusion or stiff reactions are present.

Key words: Evolutionary Advection-Diffusion-Reaction Partial Differential Equations, Approximate Matrix Factorization, Runge-Kutta Radau IIA methods, Stability.