Conference in Numerical Analysis 2014 (NumAn 2014)
September 2-5, 2014
Chania,Greece

Successive approximations for optimal control in some nonlinear systems with small parameter

Alexander Spivak

Department of Computer Science, HIT, Holon Institute of Technology,
52 Golomb str., Holon, ISRAEL
spivak@hit.ac.il

Abstract

An important class of nonlinear control systems is bilinear systems. Such systems are linear on phase coordinates when the control is fixed, and linear on the control when the coordinates are fixed. The first point for the study of bilinear systems is to investigate the dynamic processes of nuclear reactors, kinetics of neutrons, and heat transfer. Further investigations show that many processes in engineering, biology, ecology and other areas can be described by the bilinear systems . It is shown that bilinear systems may be applied to describe some chemical reactions and many physical processes in the growth of the human population.

In this work we consider nonlinear stochastic systems that can be described in the form

$$\dot{x}(t) = \epsilon f_1(t,x) + B(t)x(t)u(t) + \sigma \dot{w}(t),$$
$$x(0) = x_0, \;\; 0 \le t \le T. \tag{1}$$

Here the vector $x(t)$ is from the Euclidean space E_n, the control $u(t) \in E_m$, the matrices σ and B have continuous and bounded elements, $\epsilon \ge 0$ is a small parameter, and the initial vector $x_0 \in E_n$ and the constant $T \ge 0$ are given. The function $f_1(t,x) \in E_n$ is continuous in the totality of its arguments, and satisfies some constraints, $w(t)$ is standard Wiener process. The matrix $\sigma(t)$ in (1) is such that $\sigma(t)\sigma'(t)$ is positive definite. We understand the equation (1) in the sense of Ito. Note that if $\epsilon = 0$, initial system (1) is stochastic bilinear, that is, it contains a nonlinearity of the form $x(t)u(t)$. The problem is to find a control u minimizing the functional $J(0,u)$, where

$$\begin{aligned} J(t,u) \;\; &= \;\; M\Bigg[x'(t)H_1x(t) + \int_t^T \big(\, x'(s)H_2(s)x(s) + \\ &+ \;\; u'(s)H_3(s)u(s) + f(s,x(s))\big)\, ds\Bigg]. \end{aligned} \tag{2}$$

Here $H_i,\ i = 1,2,3$ are given matrices, so that H_1, $H_2(t)$ are non-negative defined, $H_3(t)$ is positive defined in the interval $[0,T]$, and the matrices $H_2(t)$ and $H_3(t)$ are measurable and bounded. The vector $f(t,x)$ is determined later.

When $\epsilon = 0$, the optimal control synthesis is found in an exact analytic form. Successive approximations to the optimal control are constructed with the help of the perturbation method. Error estimates of the suggested method are presented.

Key words: Successive Approximations, Small Parameter, Perturbation Theory.