

# Domain decomposition method with complete radiation boundary conditions for the Helmholtz equation in waveguides

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## Abstract

In this paper, we present a nonoverlapping domain decomposition algorithm with a high-order transmission condition for the Helmholtz equation posed in a waveguide. We introduce the new high-order transmission conditions based on the complete radiation boundary conditions (CRBCs) that have been developed for high-order absorbing boundary conditions [3, 4]. We verify the rapid convergence of the Schwarz algorithm in terms of the order of CRBCs. It will be shown that damping parameters involved in the transmission conditions can be selected in an optimal way for enhancing the convergence of the Schwarz algorithm. This algorithm can also be employed efficiently for a preconditioner in GMRES implementations as recently developed sweeping preconditioners [1, 2, 5]. Finally, numerical examples confirming the theory will be presented.

*Key words:* Helmholtz equation, domain decomposition, complete radiation boundary condition.

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