Conference in Numerical Analysis 2014 (NumAn 2014)
September 2-5, 2014
Chania,Greece

A recursive multilevel approximate inverse-based preconditioner for solving general linear systems

Yi-Ming Bu[a,b] **and Bruno Carpentieri**[b]

[a]School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, China

[b]Institute of Mathematics and Computer Science, University of Groningen, Groningen, The Netherlands

y.bu@rug.nl,b.carpentieri@gmail.com

Abstract

We consider multilevel approximate inverse-based preconditioning techniques for solving systems of linear equations

$$Ax = b \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ is a typically large, sparse, nonsymmetric matrix arising from the discretization of partial differential equations. Approximate inverse methods directly approximate A^{-1} as the product of sparse matrices, so that the preconditioning operation reduces to forming one (or more) sparse matrix-vector product. Due to their inherent parallelism and numerical robustness, this class of methods is receiving renewed consideration for iterative solutions of large linear systems on emerging massively parallel computer systems. In practice, however, some questions need to be addressed. First of all the computed preconditioner could be singular. In the second place, these techniques usually require more CPU-time to compute the preconditioner than ILU-type methods. Third, the computation of the sparsity pattern of the approximate inverse can be problematic, as the inverse of a general sparse matrix is typically fairly dense. This leads to prohibitive computational and storage costs.

We present an algebraic recursive multilevel inverse-based preconditioner, based on a distributed Schur complement formulation, that attempts to remedy these problems. The proposed solver uses recursive combinatorial algorithms to preprocess the structure of A and to produce a suitable permutation of the linear system that can maximize sparsity in the approximate inverse. An efficient tree-based recursive data structure is generated to compute and apply the approximate inverse fast and efficiently. We report on numerical experiments on matrix problems arising in different application areas to illustrate the potential of the proposed solver to reduce significantly the number of iterations of Krylov methods at low memory costs, also compared to other sparse approximate inverses and multilevel Schur-complement based incomplete LU factorization methods and software. Finally, we discuss block generalizations of our method that can exploit available block structure in the matrix to maximize computational efficiency.

Key words: Krylov subspace methods, Approximate inverse preconditioners, combinatorial algorithms.