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Numerical techniques for sliding motion in Filippov discontinuous systems

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Abstract

In this talk we present numerical techniques to approximate the solution of a discontinuous differential system of Filippov type during *sliding motion*. Namely, for a given surface Σ defined as the 0-set of a smooth scalar function h: $\Sigma = \{x : \ h(x) = 0\}$, we have the problem $x' = f(x)$, where $f(x) = f_1(x)$ when $h(x) < 0$, and $f(x) = f_2(x)$ when $h(x) > 0$. Further, $(\nabla h)^T f_1 > 0$ and $(\nabla h)^T f_2 < 0$, on and near Σ, so that trajectories are attracted to Σ and must remain there.

So the main steps of a numerical procedure for solving such kind of problems are: reach Σ at x_0 (by an event location procedure) and, starting with $x_0 \in \Sigma$, solve the differential system $x' = (1-\alpha)f_1 + \alpha f_2$, where α has to be found so that x' is tangent to Σ: *sliding motion*.

Here we propose an event location procedure, which determines the event point on Σ in a finite number of steps, and compare different numerical procedures to integrate our piecewise differential system during the sliding motion.

It is well understood that when one integrates the differential system on Σ typically the numerical solution does not remain on Σ. Thus, the main feature of an effective numerical method is to require that the the numerical solution also remains on Σ. To achieve this, projection techniques can be used.

We will consider the following three different flavors of projection techniques.

(i) Classical projection of the numerical solution obtained by any method, say an explicit method. In particular, we will discus two ways to perform this projection.

(ii) A change of variable technique, in case one can write explicitly $h(x) = 0 \Leftrightarrow x_k = g(x_1, \ldots, x_{k-1}, x_{k+1}, \ldots, x_n)$.

(iii) Reverse projection technique, whereby rather than starting with x_0 on Σ we seek a perturbed initial condition $\tilde{x}_0 \approx x_0$ such that the value of x_1 computed by one step of a numerical method starting at $\tilde{x}_0$ is on Σ. Even for this technique, we will present different implementations.

We will compare the above projection techniques on several examples.

Key words: Discontinuous ODEs, Filippov systems, sliding motion, one-step methods, projection methods.