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Method for solving degenerate sub-definite nonlinear equations

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Abstract

We consider system of nonlinear equations

$$F(x) = \begin{bmatrix} f_1(x) \\ ... \\ f_m(x) \end{bmatrix} = 0,$$

where $x \in \mathbf{R}^n$, $m \leq n$, and $F'(x^*)$ is degenerate at the solution point x^*.
Denote the feasible set as follows $M(x^*) = \{x \in \mathbf{R}^n | F(x) = 0\}$.

It is known that in nondegenerate case (when $F'(x^*)$ is nondegenerate) the Newton-Gauss method converges to some point $\tilde{x}^* \in M(x^*)$([1] pp. 228), and the rate of convergence is quadratic.

In the complete system of nonlinear equations one of the main result of the p- regularity theory is p-factor method. Scheme of this method is as follows

$$x_{k+1} = x_k - \{\psi_p'(x_k)\}^{-1}\psi_p(x_k),$$

where p-factor operator $\psi_p(x_k)$ has following form

$$\psi_p(x_k) = P_1F'(x_k) + P_2F''(x_k)[h] + ... + P_pF^{(p)}(x_k)[h]^{[p-1]}.$$

Here P_1 is ortoprojection onto Im$(F'(x^*))^{\perp}$ and $P_i, i = 2, ..., p$ also ortoprojection on the same defined sets and the element $h(\|h\| = 1)$ we construct in such a way that p-factor matrix

$$P_1F'(x^*) + P_2F''(x^*)h + ... + P_pF^{(p)}(x^*)[h]^{[p-1]}$$

is nonsingular at the solution point $x^* = 0$ (p-regular along h).

However for degenerate sub-definite nonlinear equations has been considered only the case for $p = 2$. In this paper we present the generalization of the p-factor method for sub-definite case and $p \geq 2$. We will also describe a numerical algorithm of the p-factor method and will give numerical results.

References

[1] A.F. IZMAILOV, A. A.TRET'YAKOV, *Factor-analysis of nonlinear mappings*, Nauka, Moscow, 1994.[in Russian]

Key words: nonlinear equation, p-factor operator, p-regularity, singularity, necessary and sufficient conditions, nonregular constraints, p-factor methods