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On positivity preservation for finite element based methods for the heat equation

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Abstract

We consider the model initial–boundary value problem

$$u_t - \Delta u = 0, \text{ in } \Omega, \quad u = 0, \text{ on } \partial\Omega, \quad \text{for } t \geq 0, \quad u(0) = v, \text{ in } \Omega, \tag{1}$$

where Ω is a bounded convex polygonal domain in $\mathbb{R}^2$. By the maximum-principle, we have

$$v \geq 0 \text{ in } \Omega \quad \text{implies} \quad u(t, \cdot) \geq 0 \text{ in } \Omega, \text{ for } t \geq 0. \tag{2}$$

Our purpose is to discuss analogues of this property for some finite element methods, based on piecewise linear finite elements, including, in particular, the Standard Galerkin (SG) method, the Lumped Mass (LM) method, and the Finite Volume Element (FVE) method.

We consider the analog semidiscrete problem of (1), where we discretize space using either the SG, LM or FVE method. It is known that for the semidiscrete SG the analog of (2) does not hold for all $t \geq 0$. However, in the case of the LM method, this holds if and only if the triangulation is of Delaunay type. For the FVE method we will show here that the situation is the same as for the SG method.

However, when the solution is not positive for all $t > 0$, it may be positive for all t sufficiently large. We shall study this and approximate a corresponding minimum t_0 such that for all $t > t_0$ the solution of the semidiscrete problem is positive. Also we consider similar results for the corresponding fully discrete problems, when we discretize time with the backward Euler method. Finally we provide numerical results in 1 and 2 dimensions.

Key words: positivity, finite element method, finite volume method, lumped mass method, Delaunay triangulation.