Conference in Numerical Analysis 2014 (NumAn 2014)
September 2-5, 2014
Chania,Greece

Chebyshev accelerated preconditioned MHSS iteration methods for a class of block two-by-two linear systems

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Abstract

The preconditioned modified Hermitian and skew-Hermitian iteration method [1] is efficient for solving the the following block two-by-two systems of linear equations

$$\mathbf{A}\mathbf{x} \equiv \begin{pmatrix} \mathbf{W} & -\mathbf{T} \\ \mathbf{T} & \mathbf{W} \end{pmatrix} \begin{pmatrix} \mathbf{y} \\ \mathbf{z} \end{pmatrix} = \begin{pmatrix} \mathbf{p} \\ \mathbf{q} \end{pmatrix} \equiv \mathbf{g}.$$

It could be written as the following procedure:

$$\begin{cases} \begin{pmatrix} \alpha\mathbf{V}+\mathbf{W} & 0 \\ 0 & \alpha\mathbf{V}+\mathbf{W} \end{pmatrix} \begin{pmatrix} \mathbf{y}^{(k+\frac{1}{2})} \\ \mathbf{z}^{(k+\frac{1}{2})} \end{pmatrix} = \begin{pmatrix} \alpha\mathbf{V} & \mathbf{T} \\ -\mathbf{T} & \alpha\mathbf{V} \end{pmatrix} \begin{pmatrix} \mathbf{y}^{(k)} \\ \mathbf{z}^{(k)} \end{pmatrix} + \begin{pmatrix} \mathbf{p} \\ \mathbf{q} \end{pmatrix}, \\ \begin{pmatrix} \alpha\mathbf{V}+\mathbf{T} & 0 \\ 0 & \alpha\mathbf{V}+\mathbf{T} \end{pmatrix} \begin{pmatrix} \mathbf{y}^{(k+1)} \\ \mathbf{z}^{(k+1)} \end{pmatrix} = \begin{pmatrix} \alpha\mathbf{V} & -\mathbf{W} \\ \mathbf{W} & \alpha\mathbf{V} \end{pmatrix} \begin{pmatrix} \mathbf{y}^{(k+\frac{1}{2})} \\ \mathbf{z}^{(k+\frac{1}{2})} \end{pmatrix} + \begin{pmatrix} \mathbf{q} \\ -\mathbf{p} \end{pmatrix}, \end{cases}$$

where α is a given positive constant and $\mathbf{V} \in \mathbb{R}^{n\times n}$ is a prescribed symmetric positive definite matrix. The Chebyshev semi-iteration method is fulfilled for accelerating the above iteration method. It could be verified that the Chebyshev accelerated PMHSS iteration method is a parameter free method. It converges unconditionally. The new method is utilized on solving the distributed control problems. Numerical experiments shows that the performance of the Chebyshev accelerated PMHSS iteration method is independent on not only the mesh size and the regularization parameter of the cost functional.

Key words: Chebyshev semi-iteration, PMHSS iteration, PDE-constrained optimization, block two-by-two matrices.

References

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