**Conference in Numerical Analysis 2014 (NumAn 2014)**
**September 2-5, 2014**
**Chania,Greece**

# Some new perturbation bounds of generalized polar decomposition

**X.-L. Hong, L.-S. Meng and B. Zheng**
School of Mathematics and Statistics, Lanzhou University
Lanzhou, Gansu Province, P.R.China
hongxiaoli2007@163.com,menglsh07@lzu.edu.cn,bzheng@lzu.edu.cn

**Abstract**

Let $A,\ \widetilde{A} = A + E \in C^{m\times n}$ have the (generalized) polar decompositions

$$A = QH \text{ and } \widetilde{A} = \widetilde{Q}\widetilde{H}, \tag{1}$$

where $Q$ is subunitary and $H$ is Hermitian positive semi-definite. We present the following new bounds of the positive (semi-)definite polar factor and the (sub) unitary polar factor for the (generalized) polar decomposition under the general unitarily invariant norm $\|\cdot\|$ and the spectral norm $\|\cdot\|_2$, which are stated as in the following theorem.

**Theorem.** Let $A, \widetilde{A} = A+E \in C_r^{m\times n}$ have the (generalized) polar decompositions in (1).

(1). When $r < n \leq m$, we have

$$\|\widetilde{H} - H\| \leq \left(2 + \frac{2\min\{\sigma_1, \widetilde{\sigma}_1\}}{\sigma_r + \widetilde{\sigma}_r}\right)\|E\|,$$

$$\|\widetilde{Q} - Q\|_2 \leq \frac{1}{\sigma_r + \widetilde{\sigma}_r}\left(1 + \sqrt{1 + \frac{\sigma_r^2 + \widetilde{\sigma}_r^2}{\max\{\sigma_r^2, \widetilde{\sigma}_r^2\}}}\right)\|E\|_2.$$

(2). When $r = n \leq m$, we have

$$\|\widetilde{H} - H\| \leq \left(1 + \frac{2\min\{\sigma_1, \widetilde{\sigma}_1\}}{\sigma_n + \widetilde{\sigma}_n}\right)\|E\|,$$

$$\|\widetilde{Q} - Q\| \leq \frac{1}{\sigma_n + \widetilde{\sigma}_n}(2 + \frac{\min\{\sigma_n, \widetilde{\sigma}_n\}}{\max\{\sigma_n, \widetilde{\sigma}_n\}})\|E\|,$$

$$\|\widetilde{Q} - Q\|_2 \leq \frac{1}{\sigma_n + \widetilde{\sigma}_n}\left(1 + \sqrt{1 + \frac{\min\{\sigma_n^2, \widetilde{\sigma}_n^2\}}{\max\{\sigma_n^2, \widetilde{\sigma}_n^2\}}}\right)\|E\|_2.$$

We can show that the estimated bounds in the above theorem are sharper than the existing corresponding ones in the literatures. Futhermore, note that if $A = QH$, then

$$M^{1/2}AN^{-1/2} = (M^{1/2}QN^{-1/2})(N^{1/2}HN^{-1/2})$$

for any $M \in C_>^{m\times m}$ and $N \in C_>^{n\times n}$. Hence, all perturbation bounds in the above theorem can be naturally extended to the case of the weighted polar decomposition of $A$, which also improved the known perturbation bounds for the weighted polar decomposition.

*Key words:* Perturbation bounds; Positive semi-definite polar factor; Subunitary polar factor; Generalized polar decomposition; Weighted polar decomposition; Unitarily invariant norm; Spectral norm.