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Method for solving nonlinear singular problems

Agnieszka Prusińska[a] **and Alexey A. Tret'yakov**[b,c]
[a]Department of Mathematics and Physics, Siedlce University of Natural Sciences and Humanities, Siedlce, Poland
[b]System Research Institute, Polish Academy of Sciences
Warsaw, Poland
[c]Computing Center, Russian Academy of Sciences,
Moscow, Russia
aprus@uph.edu.pl, tret@uph.edu.pl

Abstract

The aim of our work is to investigate conditions for existence of local solutions to nonlinear equations of the form

$$F(x) = 0, \quad F : \mathbf{R}^n \to \mathbf{R}^m, \tag{1}$$

in degenerate case, i.e. when $\operatorname{Im} F'(x_0) \neq \mathbf{R}^m$ and x_0 is a chosen initial point. We propose an algorithm of the numerical method that is convergent to a solution point in the mentioned case. In our approach we use p-factor operator and some elements of p-regularity theory [1]. Main result of this theory is a description of the tangent cone to the solution set in degenerate case.

We assume that F is a $p+1$ times differentiable mapping and for some $h \in \mathbf{R}^n$ consider the sequence

$$x_{k+1} = x_k - \Lambda_h^{-1} \left(f_1(x_0 + \omega h + x_k), \dots, f_p(x_0 + \omega h + x_k) \right), \quad k = 1, 2, \dots, \tag{2}$$

where $0 < \omega < 1/2$. An operator $\Lambda_h = \left(f_1'(x_0), f_2''(x_0)[h], \dots, \frac{1}{(p-1)!} f_p^{(p)}(x_0)[h, \dots, h] \right)$ for $x \in \mathbf{R}^n$ is called p-factor operator. To construct this operator we decompose $\mathbf{R}^m$ into a direct sum $Y_1 \oplus \dots \oplus Y_p$ and define auxiliary mappings $f_i : \mathbf{R}^n \to Y_i$, $f_i(x) = P_{Y_i} F(x)$, $i = 1, \dots, p$, such that $f_i^{(k)}(x_0) = 0$, $k = 1, \dots, i-1$ and $P_{Y_i} : \mathbf{R}^m \to Y_i$ – the projection operator, where Y_i is closed subspace of Y (see [1]). For a linear operator Λ_h we define its right inverse Λ_h^{-1} and $\Lambda_h^{-1} y$ is an element $x \in \mathbf{R}^n$ such that $\|x\| = \min \{\|z\| : \Lambda_h(z) = y\}$. By the "norm" of Λ_h^{-1} we mean the number $\|\Lambda_h^{-1}\| = \sup_{\|y\|=1} \inf\{\|x\| : \Lambda_h x = y, \ x \in \mathbf{R}^n\}$. If $\operatorname{Im} \Lambda_h = \mathbf{R}^m$, then the sequence (2) is convergent to the solution of (1).

We illustrate the basic idea of the method with some numerical examples.

Key words: p-regularity, singularity, contracting mapping, multimapping, p-factor operator.

References

[1] Tret'yakov A. A., Marsden J. E.: Factor-analysis of nonlinear mappings: p-regularity theory, Commun. Pure Appl. Math. 2, 425–445 (2003)