Conference in Numerical Analysis 2014 (NumAn 2014)
September 2-5, 2014
Chania,Greece

Numerical stability of block direct methods for solving symmetric saddle point problem

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Abstract

We study the numerical properties of some block direct methods for solving the following saddle point problem (quasidefinite case)

$$Mz = f \Leftrightarrow \begin{pmatrix} A & B \\ B^T & -C \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b \\ c \end{pmatrix}, \tag{1}$$

where $A \in R^{m\times m}$, $C \in R^{n\times n}$ are symmetric positive definite and $B \in R^{m\times n}$, $n \leq m$. Then M is nonsingular and there is a unique solution (x_*, y_*) of (1). Such problems arise in many applications, e.g., in optimization, in the solution of PDEs, weighted least squares (image restoration), FE formulations of consolidation problem. The matrices A and B are usually large, sparse and ill-conditioned. Structure of the problem leads to the application of block methods which operate on groups of columns of M and allow to apply the BLAS-3 compatible algorithms. It is known that the block LU methods are not numerically stable, in general. The block factorization

$$M = \begin{pmatrix} A & B \\ B^T & -C \end{pmatrix} = \begin{pmatrix} I & 0 \\ B^T A^{-1} & I \end{pmatrix} \begin{pmatrix} A & 0 \\ 0 & -(C + B^T A^{-1} B) \end{pmatrix} \begin{pmatrix} I & A^{-1}B \\ 0 & I \end{pmatrix}$$

is commonly used to solve the equation (1). We analyze the methods avoiding computing the Schur complement $S = -(C + B^T A^{-1} B)$. If C is ill-conditioned then the computed Schur complement S may be singular in working precision. We propose and analyze algorithms for solving symmetric saddle point problem which are based upon the block Cholesky decomposition and the block Gram-Schmidt method. In particular, we prove that the algorithm BCGS2 (Reorthogonalized Block Classical Gram-Schmidt) using Householder Q-R decomposition implemented in the floating point arithmetic is backward stable, under a mild assumption on the matrix M.

Extensive numerical testing was done in MATLAB to compare the performance of some direct methods for solving linear system of equations of special block matrices.

Key words: symmetric quasidefinite (sqd) systems, saddle point problem, QR decomposition, numerical stability, condition number, iterative refinement.