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Nyström methods for two-dimensional Fredholm integral equations on unbounded domains

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Abstract

We investigate the numerical solution of two-dimensional Fredholm integral equations defined on the set $S = [a, b] \times [c, d]$, $-\infty \leq a < b \leq \infty$, $-\infty \leq c < d \leq \infty$,

$$f(x, y) - \mu \int_S k(x, y, s, t) f(s, t) \mathbf{w}(s, t)\, ds\, dt = g(x, y), \quad (x, y) \in S, \tag{1}$$

where $\mathbf{w}(x, y) := w_1(x)w_2(y)$ and w_1, w_2 are suitable weight functions defined on $[a, b]$, $[c, d]$ respectively, μ is a real number. k and g are given functions defined on $([a, b] \times [c, d])^2$ and $[a, b] \times [c, d]$ respectively, which are sufficiently smooth on the open sets but can have (algebraic) singularities on the finite boundaries and an exponential growth at $\pm\infty$ at most w.r.t. each variable. f is the unknown function.

Therefore S is intended to be an unbounded domain, for instance a quarter of the plane, a strip etc.

We introduce some Nyström methods based on cubature formulas obtained as tensor products of two Gaussian quadrature formulas w.r.t. the weights w_1, w_2. Due to the "unboundedness" of the domain we need to "truncate" the quadrature rules. The convergence, stability and well conditioning of the methods are proved in suitable weighted spaces of continuous functions. Some numerical examples illustrate the efficiency of the methods.

Key words: Fredholm integral equation, Nyström method, spectral methods