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Efficient GPU pricing of interest rate derivatives: PDE formulation and ADI methods

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Abstract

We study the parallel implementation on a Graphics Processing Unit (GPU) of Alternating Direction Implicit (ADI) time discretization methods for solving three-dimensional time-dependent parabolic Partial Differential Equations (PDEs) with mixed spatial derivatives, and investigate the performance of the resulting parallel methods in pricing foreign exchange (FX) interest rate hybrids, namely Power Reverse Dual Currency (PRDC) swaps with various exotic features.

A model for pricing PRDC swaps involves three stochastic factors, namely the FX rate, and the interest rates in the two currencies. By certain financial and mathematical arguments, the resulting model is a three-dimensional in space parabolic PDE, which includes all cross-derivative terms and, assuming a local volatility model, has variable coefficients. We use standard centered Finite Differences (FDs) for the space discretization and the Hundsdorfer-Verwer (HV) ADI method for the timestepping. We discuss the parallelization on a GPU of the computational requirements of the ADI method, such as the multiple tridiagonal solutions along each of the problem's spatial dimensions and the matrix-vector products, with special attention to coalesced memory access.

Furthermore, we consider the highly popular Target Redemption (TARN) feature for PRDC swaps, which adds path-dependency and, therefore, complexity to the PDE problem. The pricing of the FX-TARN PRDC swap is handled by breaking it down into several independent pricing subproblems over each period of the tenor structure. Each of the subproblems is solved on an individual GPU, with communication at the end of each period of the tenor structure taken care by MPI.

We present numerical experiments that indicate considerable speedup, when comparing the CPU versus the GPU implementations, as well as the implementations on one versus multiple GPUs.

Key words: Power Reverse Dual Currency (PRDC) swaps, local volatility, Target Redemption (TARN), three-dimensional parabolic PDE, Hundsdorfer-Verwer ADI method, Graphics Processing Unit (GPU), MPI.