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Numerical evaluation of hypersingular integrals on the semiaxis

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Abstract

We consider hypersingular integrals of the following type

$$\mathcal{H}_p(g,t) = \int_0^{+\infty} \frac{g(x)}{(x-t)^{p+1}} dx,$$

where $0 < t < +\infty$, $p \geq 1$ is an integer and the function g behaves like x^α for $x \to 0$ and has an exponential or algebraic decay for $x \to +\infty$, i.e., it can be written in one of the following forms

- $g(x) = f(x) w_\alpha(x), \quad w_\alpha(x) = x^\alpha e^{-x}, \alpha \geq 0;$
- $g(x) = f(x) u_{\alpha,\beta}(x), \quad u_{\alpha,\beta}(x) = \frac{x^\alpha}{(1+x)^\beta}, \alpha, \beta \geq 0.$

They appear in different contexts and, in particular, in some problems of mathematical theory of elasticity (see [2]) and in hypersingular integral equations coming from Neumann two-dimensional elliptic problems defined on a half-plane by using a Petrov-Galerkin infinite BEM approach as discretization technique (see [1]). To our knowledge, the literature dealing with the approximation of hypersingular integrals on unbounded intervals is very poor.

In this talk we propose some numerical procedures for the pointwise approximation of the integrals $\mathcal{H}_p(g,t)$ that are based on Gaussian-type quadrature formulas. We prove that such procedures are stable and convergent in suitable weighted uniform spaces and, for each of them, we give error estimates. Finally, we show that the theoretical results are confirmed by the numerical tests.

Key words: hypersingular integrals, Gaussian-type quadrature rules.

References

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