Conference in Numerical Analysis 2014 (NumAn 2014)
September 2-5, 2014
Chania,Greece

A 1D stabilized finite element model for non-hydrostatic wave breaking and run-up

Paola Bacigaluppi [a] **and Mario Ricchiuto**[b]

[a] Department of Mathematics, Universität Zürich,
Zürich, Switzerland

[b] Inria Bordeaux Sud-Ouest,
Talence cedex, France

paola.bacigaluppi@gmail.com,Mario.Ricchiuto@inria.fr

Abstract.
A new methodology is presented to model the propagation, wave breaking and run-up of waves in coastal zones. Propagation is modelled by a form of the enhanced Boussinesq equations (Madsen and Sorensen, *Coast.Eng.* 1992), while the forming of a roller in breaking regions is captured by reverting to the shallow water equations and allowing waves to locally converge into discontinuities. The switch between the two models is defined by a wave breaking criterion that depends on several physical parameters, including the shape and celerity of the wave and the presence of dry areas. To discretize the system we propose a non-linear variant of the stabilized finite element method of (Ricchiuto and Filippini, *J.Comput.Phys.* 2014). To guarantee monotone shock capturing, a technique based on a non-linear mass-lumping allows to provide local non-oscillatory approximations of discontinuities reverting from a third order scheme in smooth regions to a first order upwind scheme. The local character of the mass-lumping is guaranteed by the use of limiters, or of properly defined smoothness sensors. The presented scheme guarantees positivity preservation, well balancedness and the treatment of wet/dry fronts. The wave breaking is triggered by means of three different criteria, including a local implementation of the theoretical convective criterion of (Bjørkavåg and Kalisch, *Phys.Letters A* 2011), which have been thoroughly analysed and tested. The model obtained is validated on several benchmarks showing excellent agreement with the available experimental data. As an example the figure below shows the run-up of a periodic wave over a constant slope. In particular the solution between the black lines corresponds to the detected breaking area and is computed through the shallow water model.

Key words: Wave propagation, wave breaking, shock-capturing, stabilized finite elements, SUPG scheme, Boussinesq equations, shallow water equations, wet/dry fronts, wave breaking model.

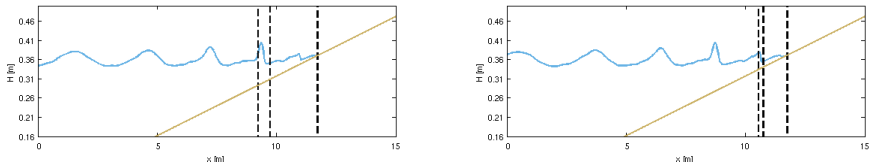

Fig.: Snapshots of the first and last breaking instants for a periodic wave run up on a constant slope. Result obtained with the local variant of the convective criteria.