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The Fokas Method and Initial-Boundary Value Problems for Multidimensional Integrable PDEs

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Abstract

The Fokas Method, or Unified Transform Method, introduced in (A. S. Fokas, Proc. Roy. Soc. Lond. A **453** (1997), 1411-1443), is the appropriate generalization of the classical inverse scattering method which renders it applicable to the far more rich context of initial-boundary value problems (IBVPs) for integrable evolution PDEs in one spatial dimension.

It is, however, interesting that there do also exist physically significant evolution equations in 2 or more spatial dimensions which share the property of integrability, i.e. that of admitting a Lax pair formulation. The most well-known integrable nonlinear PDEs in 2+1 (2 spatial and 1 temporal) dimensions are the so-called Davey-Stewartson (DS) and Kadomtsev-Petviashvili (KP) equations. Recently (A. S. Fokas, Commun. Math. Phys. **289** (2009), 957-993) took the first step towards the extension of his method to the case of multidimensions. In particular, the problem treated therein was the IBVP for the DS equation - as well as for its linearized version - posed on the half-plane. Soon thereafter, the half-plane case of the KP equation was also analyzed.

In the present work we attempt to generalize this methodology so as to cover cases of more general domains, again in the context of (2+1)-dimensional PDEs. The linearized version of the DS equation is used as a prototypical example. In particular, we analyze the following two problems:(i) the quarter-plane IBVP, and (ii) the IBVP in a rectangular domain, both problems under smooth, temporally-decaying, non-homogeneous boundary conditions.

The approach is totally based on the Lax pair formulation of the given PDE, and thus is the first step towards the construction of a formalism for the *nonlinear* case, i.e., for the DS equation itself, in each one of the geometries (i) and (ii). It is shown how both two eigenvalue equations constituting the Lax pair can undergo a simultaneous spectral analysis associated to any of the given domains (i) and (ii). Thus, in any one of the two cases, we achieve an appropriate d-bar problem for a sectionally non-analytic (in fact, sectionally generalized-analytic) function, i.e., for a function that has different generalized-analytic representations in different regions of the complex plane.

In the presentation, if time permits, we will also briefly refer to the nonlinear case, i.e., to the DS equation itself in each one of the geometries (i) and (ii).

Key words: Lax pairs, initial-boundary value problems, multidimensions.