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Essential spectral equivalence via multiple step preconditioning and applications to ill conditioned Toeplitz matrices

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Abstract

We are concerned with the fast solution of Toeplitz linear systems with coefficient matrix $T_n(f)$, where the generating function f is nonnegative and has a unique zero at zero of any real positive order θ. As preconditioner we choose a matrix $\tau_n(f)$ belonging to the so-called τ algebra, which is diagonalized by the sine transform associated to the discrete Laplacian. In previous works, the spectral equivalence of the matrix sequences $\{\tau_n(f)\}_n$ and $\{T_n(f)\}_n$ was proven under the assumption that the order of the zero is equal to 2: in other words the preconditioned matrix sequence $\{\tau_n^{-1}(f)T_n(f)\}_n$ has eigenvalues, which are uniformly away from zero and from infinity. Here we prove a generalization of the above result when $\theta < 2$. Furthermore, by making use of multiple step preconditioning, we show that the matrix sequences $\{\tau_n(f)\}_n$ and $\{T_n(f)\}_n$ are essentially spectrally equivalent for every $\theta > 2$, i.e., for every $\theta > 2$, there exist m_θ and a positive interval $[\alpha_\theta, \beta_\theta]$ such that all the eigenvalues of $\{\tau_n^{-1}(f)T_n(f)\}_n$ belong to this interval, except at most m_θ outliers larger than β_θ. Such a nice property, already known only when θ is an even positive integer greater than 2, is coupled with the fact that the preconditioned sequence has an eigenvalue cluster at one, so that the convergence rate of the associated preconditioned conjugate gradient method is optimal. As a conclusion we discuss possible generalizations and we present selected numerical experiments.

Key words: Toeplitz systems, τ algebra, preconditioning.