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Numerical Solution of the Unified Transform for Linear Elliptic PDEs in Polygonal Domains

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Abstract

Integral representations for the solution of linear elliptic partial differential equations (PDEs) can be obtained using Green's theorem. A new transform method for solving BVPs for linear and integrable nonlinear PDEs usually referred to as the *Unified Transform* or (*Fokas Transform*) was introduced by the second author [1]. The numerical implementation of this method has led to new numerical techniques for both evolution and elliptic PDEs, see for example [4, 5, 2, 3]. Here, we consider Laplace, Helmholtz, and modified Helmholtz equations in polygonal domains with a Robin boundary condition. We validate and compare the numerical solution obtained by *Unified Transform* to the solution obtained via the finite element method (FEM). We present a simple rule for choosing collocation points-i.e points in the Complex Fourier plane where the so called global relations are evaluated which guarantees a low condition number the matrix of the associated linear system.

References

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