Conference in Numerical Analysis 2014 (NumAn 2014)
September 2-5, 2014
Chania,Greece

Normalizations of the Proposal Density in Markov Chain Monte Carlo Algorithms

Antoine Zambelli[a]

[a]Anderson School of Management, University of California Los Angeles, Los Angeles, CA, USA

antoine.zambelli.2014@anderson.ucla.edu

Abstract

We explore the effects of normalizing the proposal density in Markov Chain Monte Carlo algorithms, in the context of a nonlinear inverse problem. Our problem is that of reconstructing the conductivity term K in the 2-dimensional heat equation

$$u_{xx} + u_{yy} = \frac{2H}{K\delta}u \tag{1}$$

given temperatures at the boundary points, given by d. A Metropolis-Hastings MCMC algorithm is implemented to do so. Markov Chains produce a probability distribution of possible solutions conditional on the observed data. We generate a candidate solution K' and solve the forward problem, obtaining d'. In this way, at step n the probability of setting $K_{n+1} = K'$ is given by

$$\alpha(K'|K_n) \equiv \min\left\{1, \frac{P(K'|d)g(K_n|K')}{P(K_n|d)g(K'|K_n)}\right\} \tag{2}$$

For our given proposal density g, this is initially computed as

$$\alpha = \min\left\{1, e^{\frac{-1}{2\sigma^2}\sum_{i,j=1}^{n,m}\left[\left(d_{ij}-d'_{ij}\right)^2-\left(d_{ij}-d_{n_{ij}}\right)^2\right]}\right\} = \min\left\{1, e^{-D}\right\} \tag{3}$$

We identify certain issues with this construction, stemming from large and fluctuating values of D. Using this framework, we develop normalization terms z_0, z and parameters λ that preserve the inherently sparse information at our disposal, rewriting (3) as

$$\alpha = z_0 e^{-\lambda z D} \tag{4}$$

We examine the results of this variant of the MCMC algorithm on the reconstructions of several 2-dimensional conductivity functions.

Key words: Ill-posed, Inverse Problems, MCMC, Normalization, Numerical Analysis.