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Two numerical implementations of the Fokas method for elliptic equations in a polygon

Kevin Crooks[a]
[a]DAMTP, University of Cambridge,
United Kingdom
k.m.crooks@maths.cam.ac.uk

Abstract

We consider the Dirichlet boundary value problem for the Helmholz and modified-Helmholz equations in a convex polygonal domain. Recent work has used the Fokas method to derive a Dirichlet to Neumann map for Laplace's equation on the polygon: given Dirichlet data this map recovers our unknown Neumann data. These data are coupled by an integral equation known as the *global relation*. By reformulating the global relation as a linear operator equation of the form $T\Phi = \Psi$, it was shown that T is a semi-Fredholm operator between Banach spaces. Analogous results may be obtained for the class of Helmholz equations by considering the resulting linear operators as perturbations, T_β, of T.

We analyse two numerical implementations: a Galerkin method and a pointwise method. For β not an eigenvalue for a domain, we use these approaches to solve the Dirichlet to Neumann map. Secondly, we demonstrate their use to search for eigenvalues, with the pointwise method seen to be particularly effective. Finally we discuss the numerical accuracy and difficulty of these implementations.

Key words: Fokas Method, Helmholz, Boundary Value Problems, Numerical Approach.