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Dynamical and statistical behavior of the Fermi-Pasta-Ulam model with long-range interactions

H. Christodoulidi, L. Cirto, T. Bountis and C. Tsallis
Center for Research and Applications of Nonlinear Systems (CRANS),
Department of Mathematics, University of Patras, GR–26500, Patras, Greece
hchrist@master.math.upatras.gr

Abstract

In this talk we describe the results of a recent study on a long-range-interaction generalisation of the one-dimensional Fermi-Pasta-Ulam (FPU) $\beta-$ model. In particular, we have used a coupling constant of the quartic interactions that decays as $1/r^{\alpha}$ and controls the range of interaction ($\alpha \geq 0$). We demonstrate that: (i) For $\alpha \geq 1$ the maximal Lyapunov exponent remains finite and positive for increasing number of oscillators N whereas, for $0 \leq \alpha < 1$, it asymptotically decreases as $N^{-\kappa(\alpha)}$; (ii) The distribution of time-averaged velocities is Maxwellian for α large enough, whereas it is well approached by a q-Gaussian, with the index $q(\alpha)$ monotonically decreasing from about 1.5 to 1 (Gaussian) when α increases from zero to close to one. To achieve these results for very large numbers of particles and very long integration times we made use of a number of numerical methods and strategies which will be discussed in the present talk.

Key words:
q–Gaussian Distributions, Lyapunov Exponents, Hamiltonian Lattices, Long Range Dynamics, Symplectic Integrators