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The Error Analysis of the Indirect Padé Method for Matrix Exponential [1]

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Abstract

One of the most frequently discussed matrix function is the matrix exponential. Compared with other methods for computing the matrix exponential, the scaling and squaring method highlights itself not least for its implementation in MATLAB function `expm`. Najfeld and Havel in [Adv. in Appl. Math., 16 (1995)] presented an efficient algorithm using the scaling and squaring method as well as indirect Padé approximation which is a little different from the traditional method for the computation of the matrix exponential. This method is known for its lower computation cost, however, as pointed out by Higham in [SIAM J.Matrix Anal. Appl., 26 (2005)], it is lack of sufficient error analysis. In this paper we give an analysis of the sensitivity and conditioning of the matrix polynomial function $H(B)$ by Najfeld and Havel given and conclude that $H(B)$ is a well-conditioned matrix. We also present the relative error bounds of the approximating function $(H_{2m}(B) + B)(H_{2m}(B) - B)^{-1}$ and exploit the impact of the condition number of $H(B) - B$ and the scaling times d on the relative error bounds in a heuristic way. Our numerical result shows that the algorithm given by Najfeld and Havel generally provides accuracy almost the same as the MATLAB 7.6 functions `expm` and `funm` with a lower cost and proves to be stable.

Key words: Matrix exponential, error analysis, Padé approximation, scaling and squaring, overscaling, MATLAB.

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